



ENCE HALL OF SCIENCE . UNIVERSITY OF CALIFORNIA AT BERKELEY ... AND ONLY

# What is FAMILY MATH?

It's parents and kids enjoying and learning math together.

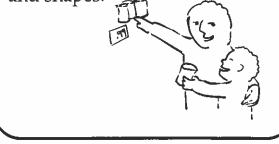


It's doing activities and playing games.

It's using beans, buttons, pennies, and toys to solve math problems.



It's being friendly with numbers and shapes.



It's exploring shapes and geometry.



It's working and talking with others.



It's estimating with numbers and sizes.



It's learning how math connects with real life.



# Mathematics In Your Home

As families and educators, most of us know that reading aloud to our children on a regular basis provides a strong foundation for success and enjoyment in reading.

But when it comes to mathematics, many of us don't know where to start. Do we buy flash cards to teach them the basic facts? Do we make them memorize rules? How do you feel now when your children ask you for help?



Do you remember your own experiences in mathematics? Was it fun or painful? Did you feel successful? In high school did you continue to take math after it was not required?

This book is for you and your child. It is about loving mathematics. It is about learning with your child, and, if you like, learning with other families.

Whether you do the activities with your child or with a group, these activities are meant to be fun. There is no pressure to rush toward right answers, or mastery of ideas. There's no test. You can take your time with an activity and move on to the next one whenever you or your children are ready.

The beauty of mathematics is in patterns. Math can be found in art, music, nature, dance, textiles, and many other everyday things and events.

### Doing Mathematics At Home

These ideas will help you create a warm and nurturing environment for mathematical explorations for your family. You will see them repeated throughout the book in different ways.

Believe that your children can succeed and let them know it. Model persistence and not giving up. We teach by what we do. Our children emulate us.

Talk with your child and really listen to what they are saying. If you do not know an answer to a problem, ask your child questions about it. You will find questions to help you do this throughout the book.

Problem-solving processes are as critical as the answers. Knowing *how* to find an answer without giving up is a lifetime skill. Getting the correct answer to a problem is not as important as the behaviors surrounding it.

Practice being a keen observer with your children whenever possible. Notice patterns on fabrics, details in plants, clouds, trees, and tiles.

Encourage your child to exchange ideas with their peers. This becomes even more important as your child grows older.

Provide a special place for study. Let your child help determine where that place will be. Making choices is part of becoming a good problem solver.

Establish positive homework habits. A consistent schedule helps children know what is expected, and they can plan around it. There is a high correlation between success in math and the amount of homework done.



# Everyday Estimation

Kids love to guess about numbers. They also want to be exactly correct. You can provide your child with many opportunities to practice this important math skill.

You can practice at the grocery store, when weighing fruits or vegetables. You can practice on the way home from school by guessing how many steps you take to cross the street, or how many minutes it takes you to drive to school.

Toys, beans, pebbles, macaroni, or other items can be used for estimating. You may need containers like jars, small plastic bags, or any see-through container you may have around the house.

# Here are some possibilities:

- Δ A small jar filled with about ten large marshmallows (don't squeeze them). You can also fill a same-size jar with smaller marshmallows and compare the different amounts. You can ask younger children to compare the two jars and tell you something that is true about the jars' contents.
- Δ A plastic bag with string in it. Is the string as long as your child is tall? As you are tall?
- Δ A jar filled with large pasta (like shell macaron and a same-size jar filled with small pasta.
- Δ A large jar filled with tennis balls (up to about ten for younger children).

- Δ 100 kernels of popcorn (unpopped) and then the same popped.
- Δ Estimate the number of jelly beans in a bag before you eat any of them.
- $\Delta$  Can you think of other things to estimate?

# **Estimation Questions:**

- Δ Write your names. How many beans do you think it will take to cover the letters?
- Δ Which name took up the most beans? Are there any other names you would like to try?
- Δ How many basketballs would it take to fill your car?
- Δ How many minutes will it take to get to school or to the market from your house?
- Δ What other things can you estimate?

Creating these opportunities will improve your child's estimating skills and accuracy.







These questions will help you keep the conversation and learning going as you explore mathematics with your child. They promote thinking and further learning. You may want to add some questions of your own.

- $\Delta$  What do you suppose would happen if...?
- $\Delta$  What will you do next?
- $\Delta$  I wonder...?
- $\Delta$  How can we check to see how close your guess is?
- $\Delta$  Why do you think that?
- $\Delta$  How did you figure that out?
- $\Delta$  Do you have any ideas about how we might begin?
- $\Delta$  How can we do this differently?
- Δ Hmm-mm, I had not thought of that. Tell me more about it.
- Δ What other ways can we show that?
- $\Delta$  Tell me about your design.
- $\Delta$  How did you decide which objects go in the circle?
- $\Delta$  What other things can we find shaped like a square/circle/triangle...?
- △ What would you do with this?
- $\Delta$  Tell me how you did that.

WHOA! Don't ask these questions all at once. One or two well-placed questions go a long way toward encouraging thinking and creating deeper understanding.

# Acknowledging Your Child's Work

Let your child know that you are aware of the thoughtful work, the creative ideas, or interesting choices that have been made.

Comment on the work by making observations.

- "I see that you have lined up three triangles and two circles. Tell me about your pattern."
- "Let's share your pattern with Mom/Dad after dinner."
- "I think your solution is interesting. Let's share it with..."
- "That is a creative way of thinking about it. Leme know what you decide."
- "That is an interesting plan. Is there anything you want me to do?"
- "You changed the rules? Tell me what they are so I can play the game with you."
- "Thank you for setting the table. I see you placed a napkin on each dish."
- "Thank you for putting your socks away. I noticed you matched a yellow sock with a blusock. Can you tell me why you decided to do that?"

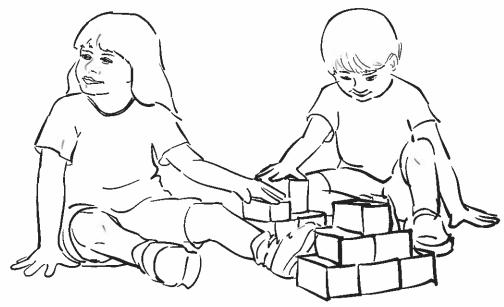
Acknowledge your child's work by describing it, rather than qualifying it by saying only that it is "neat", "wonderful", or "good". This is not to say you should not admire the work. Children want to please parents, but they also learn not to value feedback when it is always the same or without specificity.

Do ask your child's opinion of the work that has been done. This fosters accountability, self reflection, and autonomy.

Ask questions instead of making corrections (see the "sock-matching" statement above).

Accept unusual solutions, or solutions you had not expected.

Take a role in your child's activities without taking over.



# Organizing Information

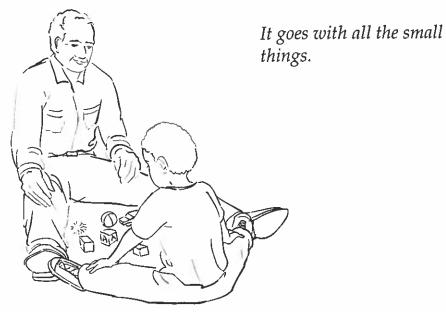
The following charts are included as suggestions for how objects might be sorted, classified, numbered, or represented. If you or your child wish to use them, copy the designs onto other paper or make photo-copies so that you can use them for many activities.

The best thing you can do for your child is to ask how the work or information might be organized. When your child finds a different way to make a set, investigate a pattern, or assign items to particular groups, ask questions about the thinking that went into the decision.

Provide alternative materials such as yarn or string for making circles or other shapes. Recycled margarine tubs, frozen food trays, and aluminum pie tins, are just a few examples of containers you can use for many of the activities in this book.

RESIST the urge to make your child do things the way you might do them. You can create a pattern, or a set, and ask your child to guess your rule. As you keep the conversation and the exploration going, your child will create other possible solution and come to a new understanding as different strategies unfold.

How did you figure out in which group the ring would go?



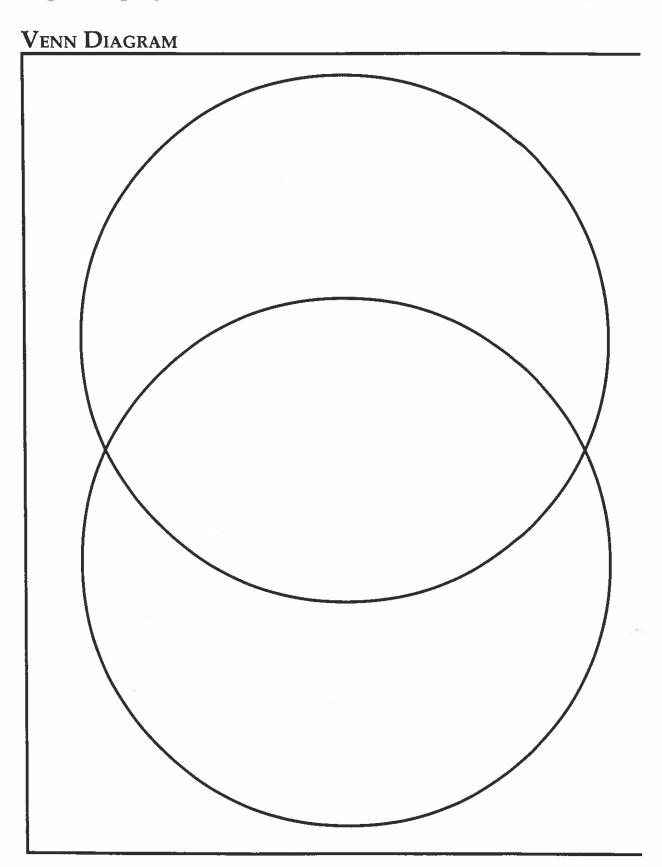
I see, what about that small key with the pennies?

It's with all the metal things, the other stuff is not made of metal.

### Insight

Organizing information helps us keep track of what has been done. Children who develop this skill will have greater success in mathematics and science. Bot h subjects require us to keep records, illustrate, and explain our work to others.

# Organizing Information (continued)



More Sorting Spaces

# Organizing Information (continued)

# BAR CHART

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# HUNDRED CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# Organizing Information (continued)

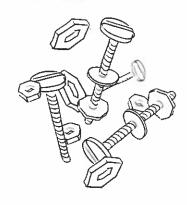
# TWENTY-FIVE CHART

				- 15-37
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

# Juts and Bolts

# HIS IS ABOUT

counting
visual reasoning
matching one to one



### OU WILL NEED

nuts and bolts of different sizes, in pairs that fit each other some beans

#### *ICTIVITY*

Empty the bag of nuts and bolts onto the table. Start with about five pairs.

sk your child to choose a nut.

- You pick a bolt that might fit it. If they fit, you get a score of 1 (or put one bean in your bowl) because you got it on the first try.
  - If they don't fit, choose another bolt and try it. If it fits, you get a score of 2 because you got it on the second try, and so forth.
- . Take turns until you have paired up all the nuts and bolts, and add up your score (beans). Keeping score is optional.
- Try the game again and compare your first score with the second one. The object is to match the nuts and bolts with the least amount of tries.
  - This is not a competitive game. The score you keep is for keeping record of your progress as a team.

#### **INSIGHT**

A you have tried this for awhile, your child will get better at matching the correct sizes. This is important in developing your child's ability in visual estimation and accuracy.

# Now I Am Tall

## This is About

∆ comparing

 $\Delta$  sizes

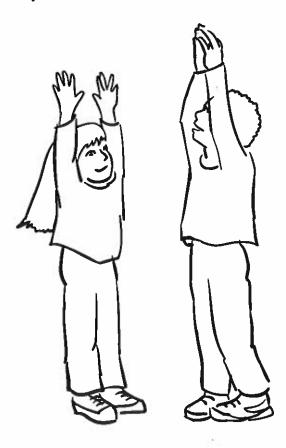
∆ shapes

## YOU WILL NEED

Δ ourselves

### **GETTING READY**

Find an indoor or outdoor space where you will be able to move freely.



Can't keep my kids sitting still too long! I've read that they need "large muscle" activities. What does that mean?

Must be like jumping and stretching and running and hopping. Mine like really BIG things to do.

What kind of things would be "not large muscle" activities?

Oh, I guess things like writing, and picking up little things, and sewing, and — well, probably TV doesn't use large muscles, either! And moving helps with ideas like short, wide, tall straight, round, and lots of other things.

#### **ACTIVITY**

Stand facing each other or side by side. Say (or sing) these comparing words, and act them out.

"Now I'm short."

"Now I'm tall."

"Now I'm wide."

"Now I'm narrow."

"Now I'm round."

"Now I'm square."

"Now my arms are sloped."

"Now they're flat."

"Now I'm straight."

"Now I'm bent."

"Now I'm standing."

"Now I'm hopping."

Make up your own comparing words.

"If I were a tree, I would be..."





# Insight

Young children should not be required to sit still for long. They need to be able to move around. It's a good idea now and then to stop "work" and do this or some other activity that lets them move all their muscles.

# Inside, Outside, On

## This is About

- $\Delta$  experience with shapes
- Δ being inside, outside, or on a shape

# YOU WILL NEED

 $\Delta$  chalk, or a long string

### GETTING READY

This is a game for two or more people.

Draw a large circle on the sidewalk or playground, or make a circle shape with string or yarn.



We're learning about shapes and positions by moving back and forth on a circle.



Why can't we just tell them what the words mean?



There's an old saying that starts out "I hear and I forget." and ends with "I do and I understand."



Oh, you mean they remember better if they act it out? That makes sense.

#### ACTIVITY

- 1. Have everybody stand just outside the circle.
- 2. Take turns giving directions, going around the circle (either clockwise or counter-clockwise—your choice!).
- 3. When a direction is given, everybody does what is said. For example:

two hops INSIDE the circle three jumps OUTSIDE the circle four slides to the left ON the circle



- 4. Think of other actions of your own, too, such as bending down, squatting, walking, clapping, or snapping fingers.
- 5. You might even try making up a chant. For example:

My name is Daniel.
Stand ON the line
and slide 4 times to the Right!

My name is Tania.
I like to jump!
Jump 3 times OUTSIDE.

# Copy-Cats

### This is About

- **∆** symmetry
- Δ spatial reasoning
- Δ using scissors

# YOU WILL NEED

- Δ some blank sheets of paper or the grid paper
- $\Delta$  paste, scissors
- δ some cut-out shapes in different colors or designs, beans, or small macaroni

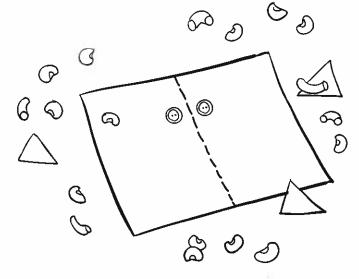
# GETTING READY

Fold your paper in half. Each partner chooses a side that will be their space.

Share the shapes or items between the two partners so that each person has an identical set of items. For example, each person should have the same kind and number of red triangles, pinto beans, elbow macaroni, and so on.

### **ACTIVITY**

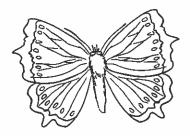
1. Have your child place a shape anywhere on her side of the paper. You copy by placing an identical shape (and color) on your side.



- 2. Continue taking turns and copying until you've taken five turns each, or until you think you have enough things on your page.
- 3. Once you think that your sides "match," paste or glue the objects to the paper.
- 4. How closely did your partner copy your design? How can you find out?
- 5. Try it again. This time you go first.

Can you and your child think of some things that already look the same on both sides? This is called "symmetry."







# INSIGHT

Be the copy-cat first. This way, you provide a model for your child. Let your child choose which work should be displayed. Ask questions about why that particular piece was chosen. This is the beginning of children looking at their own work and learning to make decisions about what they like or do not like about it.

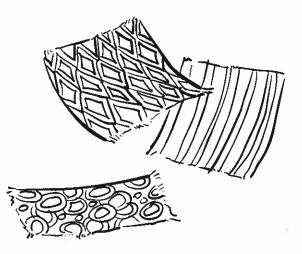
# Rag Bag

# This is About

- $\Delta$  using language in mathematics
- Δ size and texture
- Δ describing and comparing

# YOU WILL NEED

- Δ pieces of cardboard in various sizes, from small (1" by 1") to medium (up to 3" by 5")
- Δ scraps of fabric or other textured materials, such as velvet, satin, cotton, denim, sandpaper, linoleum, foil, burlap, wallpaper, and so on.
- ∆ glue
- Δ scissors
- $\Delta$  a paper bag or shoebox



Language is really important in mathematics, isn't it? We're all learning to explain our thinking.



And language helps us organize our thinking and communicate about it, just like people in most jobs.



## GETTING READY

- 1. Let the children help.
- Cut pieces of fabric or other materials to fit the cardboard pieces.
- 3. Glue the cut pieces onto the cardboard pieces, and let them dry.
- 4. If you can, make small and large pieces from the same fabric, such as a small and a large burlap, a small and a large velvet, and so on.
- 5. Put some of the completed cards into a paper bag or shoebox, ready for the activities.



After you have created the squares take time to observe the differences and similarities of the colors, textures, shapes, or other characteristics.

On the following pages you will find variations of Rag Bag.

Sometimes we might need to get started by asking each other questions like these:

What can you tell me about the size?

How does it feel to touch it?

What do you see about the shape?

What about the color?

# Rag Bag (continued)

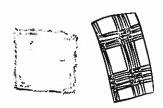
# **ACTIVITY 1: TELL IT LIKE IT IS**

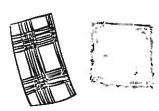
- 1. Put four or five different cards into the bag or box. Shake (gently!). Reach in (no peeking) and pull out one of the cards, then describe it.
- 2. Tell everything you notice.
- 3. Continue with the other cards in the bag, one at a time.



# **ACTIVITY 2: TWO-CARD DIFFERENCES**

- 1. Set out two cards.
- 2. Together, look at the cards and describe as many differences as you can.
- 3. Think about size, color, shape, texture (such as fuzzy or smooth), temperature (warm or cold), or even what the material was used for.
- 4. Try the same with other pairs of cards.





## **ACTIVITY 3: MATCH-UP**

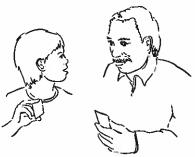
- 1. Put two or three pairs of cards that match in texture or size into the bag. Reach into the bag and pull out one of the cards. Then reach in and feel around to see if you can pull out the matching card.
- 2. Keep trying until you have a match.





## **ACTIVITY 4: DIFFERENCES**

- 1. Put about five or six cards into the bag.
- 2. Have one person reach in and pull out a card, then the other person.
- 3. Put the two cards together and tell how they are alike and how they are different.



## **ACTIVITY 5: A STORY**

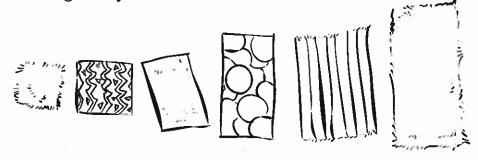
- 1. Take out your favorite card and tell a story about it.
- 2. Take out two or three cards and tell a story about their adventures.

" and then the little yellow triangle found his friend at the park..."



### **ACTIVITY 6: AN ORDERLY ROW**

- Arrange several of the cards in an orderly row, such as from smoothest to roughest, or from thickest to thinnest, or from darkest to lightest.
- 2. Make up challenges of your own.



# Share a Square Mobile

## THIS IS ABOUT

- Δ geometry
- ∆ language

## YOU WILL NEED

- $\Delta$  a straw or dowel (eight to ten inches long)
- Δ thin string or yarn
- $\Delta$  crayons or markers for coloring the squares
- $\Delta$  five or seven square pieces of construction paper, about four inches by four inches

### **ACTIVITY**

- 1. Take a square and fold it in half, then cut on the folded line. Give one half to your partner.
- 2. The second person takes a square and folds it in half in a different way and gives the other half to the other person.
- Working together, try to figure out what other shapes you can make from the remaining squares. Make two of each so that at the end you both have the same number of shapes.

What shapes did you create?

How many shapes do each of you have?

If you do not have construction paper handy, use the cardboard from cereal boxes, and cover it with recycled gift wrap or scraps of fabric.

- 4. Measure a piece of string that is three times as long as your straw and pull the string through the straw, and tie a knot in the string. (See sample below.)
- 5. Attach the shapes to different-size pieces of string and tie them on the straw so they hang in different lengths. Hang the mobile in your child's favorite spot.

# Just for Fun

Write words on the shapes that tell how you feel when you are with each other, or tell the things you enjoy doing together.



## INSIGHT

Having children trace, fold, and cutout their own shapes helps their understanding of the attributes or characteristics of the different shapes. It also helps them develop their small (fine) muscle coordination.

# **Button Boxes**

# THIS IS ABOUT

- $\Delta$  being friendly with numbers
- Δ practicing adding

# You WILL NEED

- ∆ buttons
   or small blocks
   or bottle-caps
   or design your own cardboard
   buttons
- Δ if you like, you might cut apart an egg carton to make the boxes shown

# GETTING READY

Part of the object of this activity is to practice combining small numbers to make larger numbers in an informal way, with things we can move around.

When you have played the game a few times, make up different sets of "boxes," to practice other number combinations.

With the boxes in this activity, at least two of the suggested combinations will not be possible. What are they?

Why buttons? Why can't we just write down the numbers and have the kids add them, the way we did when we were in school?





We move buttons around and match each button to its own box, so they are part of the numbers.





When do they learn about adding numbers together?





When they find that they can combine a 2-box and a 3-box to make 5, that's adding!



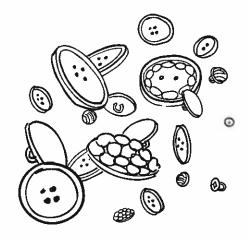


#### **A**CTIVITY

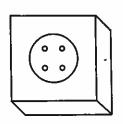
### The Story

You're helping with the button booth at the school fair. The buttons are very special. Each button must have its own space in a box. You can't sell a box with any empty spaces!

Since you can't sell a box with any empty spaces, if you put a button into one space of a box, you also must put buttons into the other spaces of that box.

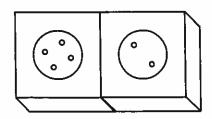


Here is a practice box. How many buttons could you put into this box?



(Only one, of course!)

Here's another practice box. How many buttons would it hold?



(two buttons)

If you used both of these boxes, how many buttons could you put in?

On the following pages is a set of boxes and a list of orders to fill. How many of the orders can you fill with those boxes?

# **Button Boxes** (continued)

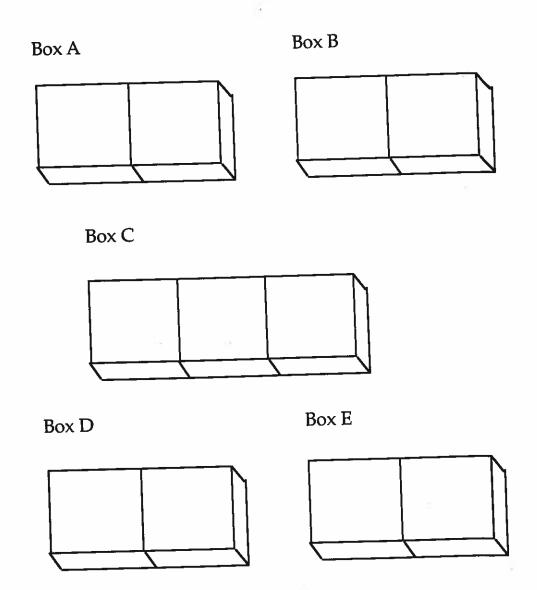
These are the button boxes you have for the button booth orders at the school fair. You may use as many boxes as you need for each order.

Talk together about each order.

Each button must have its own space.

You can't sell a box with any empty spaces.

On the next page are some button orders.

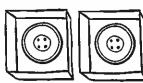




How many buttons will go in:

•	
Box A?	
Box B?	
Box C?	
Boxes A and B?	
Boxes C and D?	
Boxes C, D and E?	

Make up some of your own combinations.



Here are some orders. Which boxes would you use for these orders?

l button (can't be done!)	/ Duttoris
2 buttons	8 buttons
3 buttons	9 buttons
4 buttons	10 buttons
5 buttons	11 buttons
	12 buttons
6 buttons	

Is there more than one way to fill some of these orders?

# Button, Button, Where Is The Button?

### This is About

- Δ ordinal numbers
- Δ logic
- ∆ language

## YOU WILL NEED

- $\Delta$  ten small paper cups (small Dixie cups) or large water bottle lids
- $\Delta$  one button (needs to fit under lid or cup)
- Δ permanent marking pen or stickers of various kinds

## GETTING READY

The idea is to talk about *before* and *after* and to create strategies for making less guesses.

Younger children can do this using only three cups with stickers or shapes on them.

For older children, write the numbers on each cup, and line them up in order. You might want to start with five cups and add more as your child gets comfortable with the activity.



Try lining up the cups in a column instead of a line. Then you can use the terms *above* or *below*.

### **ACTIVITY**

- 1. One of you close your eyes (don't peek!) while the other hides a button under one of the cups.
- 2. When the button is placed, the other partner's job is to guess under which cup the button is hiding (you can look now).
- 3. For example: Your partner says: "I think the button is under the third cup." or "I think the button is under cup number three."

You say, "I put the button under a cup that comes after the third cup."

Each time talk about which cups are now eliminated. In this case it would be the third cup and all those before it.

How many guesses did you make before you guessed the correct cup?

4. Play a few rounds taking turns hiding the button.

The object is to make as few guesses as possible.

As you repeat the game, you might notice that pretty soon both of you will develop strategies for guessing the correct number with fewer clues.

What other items can you hide in the cup?

What other ways can the cups be arranged?

### INSIGHT

Some children will identify the positions as next to, over, by, or under. These are excellent and logical answers. As you and your child return to the activity, other words and understanding will evolve.

# LEAF TREASURES

Pick a leaf from the pile (one for each of you).

Compare your leaves.

Count with your partner how many pointed ends your leaf has.

How are they the same?

How are they different?

How many squares do you think your leaf will cover?

Trace your leaf on the graph paper. Count the squares your leaf covered.